



MPCS Math Learning Goals for Grades 1-8

FIRST GRADE

1. [NUMBER QUALITIES AND RECOGNITION]:

- a. Understand the qualities of numbers 1-10, finding examples in nature or human experience (e.g. two might be found in the difference between day and night, three in the sun, moon and stars, five in the star shape presented by the seeds of an apple in cross section).
- b. Recognition of written numbers 1-100.
- c. Know number names and the count sequences.
 - i. Count to 100 forwards and backwards by 1's.
 - ii. Rhythmic counting to 100 by 2's, 3's, 4's, 5's, 10's, forwards and backwards (Mastery of 2's times tables to 24, 5's to 60, 10's to 120).
 - iii. Count (forward and backward) beginning from a given number within the known sequence and ending at another given number.
- d. Reading and writing of all numerals and numbers to 20. May include introducing Roman numerals to 20.
- e. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

2. [COMPUTATIONS AND PROCEDURES]:

- a. Concept of 4 operations as qualities/images (e.g. multiplication as repeated addition, division as fair sharing, subtraction as a giving away, addition as acquisitiveness. There are many possibilities, but the general intent is to bring together numbers and feeling life).
- b. First exercises in mental arithmetic using numbers from 1-24 through story and finger (number) rhymes.
- c. Memorize the facts around the number 10 (e.g. $9+1$, $8+2$, etc.). Introduce "making ten" exercises such as $8+6 = 8+2+4$ or $9+7=10+6=16$.
- d. Use strategies such as counting on, making a ten, relating addition and subtraction, creating equivalent but easier known sums ($6+7$ can be $6+6 = 12$, plus one more).
- e. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.



MPCS Math Learning Goals for Grades 1-8

3. [GEOMETRY/FORM DRAWING]:

- a. Forms a circle or square with class. Draws a circle and square on paper.
- b. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

4. [MEASUREMENT AND DATA]:

- a. Describe measurable attributes in length or weight e.g. number of big steps to the door, a rock that is heavier than this one, a child who is shorter/taller.

SECOND GRADE

1. [NUMBER QUALITIES AND RECOGNITION]:

- a. Count forward beginning from a given number within the known sequence (instead beginning at 1)
- b. Odd and Even numbers, noting that even numbers can always be split exactly in half, (divided by two).
- c. Rhythmic counting of 3s, 4s, 6s, 7s, 8s, 9s, 11s and 12s (continued from 1st grade).
- d. Mastery of times tables in 2s, 3s, 4s, 5s, 10s, and 11s; familiarity with 6s, 7s, 8s, 9s, and 12s.
- e. Introduction to the patterns that arise in math e.g. the times tables

2. [COMPUTATIONS AND PROCEDURES]:

- a. Understand and practice reciprocal qualities of 4 processes (e.g. $3 \times 12 = 36$ and 36 divided by 3 is 12, $8 + 3 = 11$ and $11 - 3 = 8$).
- b. Further develop work with four processes using numbers up to 24 (e.g. $8 \times 3 = 24$, as does 2×12 , 4×6 etc. and likewise 24 divided by $8 = 3$, 24 divided by $2 = 12$, $19 + 5 = 24$, $24 - 5 = 19$ etc.).



MPCS Math Learning Goals for Grades 1-8

- c. Introduction and understanding of vertical and horizontal computation, with numbers all on one line (e.g. $3 \times 8 = 24$, where the “number sentence” is written the way words would be), or stacked up vertically, the singles, tens, etc. all in neat columns. The “stacking” is for addition, subtraction and multiplication. Division will take a different form in a later grade).
- d. Mastery of addition and subtraction facts up to 24.
- e. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 + \square = 11$, $5 = \square - 3$, $6 + 6 = \square$*
- f. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- g. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
- h. Introduction of place value up to the thousands e.g. ten is a bundle of ten ones.
 - i. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
 - ii. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
 - iii. Introduce carrying and borrowing (regrouping)
 - iv. Introduce read and write numbers to 1,000 using base-ten numerals, number names and expanded form
- i. Introduce greater than, less than, and equal to and their symbols
- j. Introduce multiplication of 10s (e.g. 4×20 may be thought of as 4 times two tens, which would equal 8 tens, which would equal 80).
- k. Increasing ability to do mental math story problems shared orally, using all four operations.
- l. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.



MPCS Math Learning Goals for Grades 1-8

3. [GEOMETRY AND FORM DRAWING]:

- a. Recognize symmetry and congruence through form drawing.
- b. Drawing simple shapes (circle, triangle, square, oval, etc.).
- c. Familiarity with names of simple shapes.

4. [MEASUREMENT AND DATA]:

- a. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

THIRD GRADE

1. [NUMBER QUALITIES AND RECOGNITION]:

- a. Mastery of rhythmic counting/recitation of all times tables from 1-12.
- b. Knowledge of times tables out of order, with mastery by end of 4th grade.
- c. Continued work with recognizing number patterns.

2. [COMPUTATIONS AND PROCEDURES]:

- a. Place Value
 - i. Mastery of carrying and borrowing (regrouping) with numbers with a minimum of three digits
 - ii. Mastery of reading and writing numbers to 1,000 using base-ten numerals, number names (e.g. ones, tens, hundreds etc. and expanded form (e.g. $9,836 = 9$ thousands, 8 hundreds, 3 tens and 6 ones)
 - iii. Expand place value up to millions
- b. Multiplication of multiple digit numbers by single-digit numbers up to 1,000s with carrying (e.g. $3,556 \times 6 = 21,336$).
- c. Introduction and practice of short and long division algorithms with one-place divisor (e.g. 636 divided by 6); processes to be mastered in 4th and 5th grades.



MPCS Math Learning Goals for Grades 1-8

- d. Carrying in addition and multiplication and borrowing in subtraction with numbers into the thousands place (and with zero).
- e. Daily mental math in all four processes.
- f. Continue work with greater than, less than, equal to, practice, rounding and estimation (e.g. in the question $32 \times 49 = ?$ one could round the numbers to 30 and 50, which are easy to multiply without paper. The estimate of 1,500 could then be compared to the precise answer worked out in calculation).
- g. Solve two-step word problems using the four operations. Represent these problems using equations with an empty box standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- h. Continued development of recognizing and understanding patterns in math, including addition and multiplication.
- i. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$
- j. Apply properties of operations as strategies to multiply and divide.
 - i. *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) (Students need not use formal terms for these properties)*

3. [GEOMETRY AND FORM DRAWING]:

- a. More complex symmetry and congruence through form drawing including symmetries and rotation in all four quadrants.
- b. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them using practical life stories (e.g. gardening or carpet in a room).

4. [MEASUREMENT AND DATA]:

- a. Introduce time: the calendar, telling time, and the clock with relationship to human experience (e.g. stories of the seasons, possibly measuring the length and place of one's own shadow at different times during the day with a sundial, measuring time with a candle, etc.).



MPCS Math Learning Goals for Grades 1-8

- a. Using an analog clock, tell and write time, first to the nearest hour and $\frac{1}{2}$ hour, then to the nearest five minutes and minute. Measure time intervals in minutes and hours, using a.m. and p.m.
- b. Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).
- c. Simple word problems using time and measurement.
- d. Introduce different systems of measurement; starting with ancient measurement systems using the body (e.g. cubit, a length from elbow to fingertip), and including the standard American system.
- e. Introduce and practice measurement (standard American) in length (inches, feet, yards), weight (ounces, pounds), and dry/wet volume (cups, pints, quarts, gallons) through practical work and stories (e.g. the Ark, growing plants, cooking).
- f. Introduce perimeter and area, using practical examples e.g. gardening and farming
- g. Introduce money (pennies, nickels, dimes, quarters, dollar bills) and the notation for it, practice with simple word problems (e.g. "Colored pencils cost \$.87 each, tax included. How many could you buy with \$5.00? How much change would you have left?).
- h. Collect and record data (e.g. tracking temperature, growing plants -- tied to practical activities like farming).

FOURTH GRADE

1. [NUMBER QUALITIES AND RECOGNITION]:

- a. Introduction and familiarity with fractions (e.g. numerator, denominator and the line separating them, the denominator indicating the total number of pieces and the numerator indicating the pieces one is working with at the time).
 - i. Understand a fraction as a point on the number line; represent fractions on a number line diagram (e.g. on a ruler).
 - ii. Work with equivalent fractions, recognizing common equivalent fractions like $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ through visual representations.
 - iii. Compare fractions to understand which is greater, less than or equivalent through visual pieces or drawings, recognizing comparing fractions must refer to the same whole, using like and unlike denominators.
 - iv. Express whole numbers as fractions e.g. $3 = \frac{3}{1}$, $1 = \frac{4}{4}$



MPCS Math Learning Goals for Grades 1-8

- b. Introduction and understanding of square numbers (e.g. 4, 9, 16, 25 etc.) to 144.**
- c. Introduction to prime numbers and how to determine if a number is prime or composite; knowledge of all prime numbers under 100.**
- d. Basic introduction to decimal fractions as a special type of fraction (e.g. $10/100 = .10$, which as it can be related to the notation for money introduced in third grade). Further work with decimals comes in 5th grade.**
- e. Mastery of the multiplication tables up to the 12s, out of sequence.**
- f. Mastery of place value through millions, including greater than, less than, equals, rounding (for use in estimating) and that one place is 10 times less than or greater than the digit in the next place (e.g. 700 is 10 times greater than 70).**

2. [COMPUTATIONS AND PROCEDURES]:

- a. Mastery of addition and subtraction with multi-digit numbers.**
- b. Daily mental math in all four processes.**
- c. Multiplication with carrying, with mastery of multiplication of three or more digit numbers by two-digit numbers (e.g. $423 \times 26 = 10,998$).**
- d. Practice long division (mastery in 5th), with one digit divisor, with any remainders written as fractions and with practice estimating answers before calculation.**
- e. Multiply or divide to do word problems, with multiple step problems and combining of processes (e.g. subtracting from a quantity, doubling what was left and sharing (dividing) the result five ways).**
- f. Add and subtract fractions with like denominators, including with mixed numbers.**
- g. Find multiples of a given number.**
- h. Find the least common multiple, then least common denominator (for adding and subtracting fractions with unlike denominators)**
- i. Add and subtract fractions with unlike denominators, including with mixed numbers. Introduce borrowing and carrying (regrouping) with mixed numbers.**
- j. Find the factors for any given number. Find the factor pairs for a whole number between 1-100.**



MPCS Math Learning Goals for Grades 1-8

- k. Find the greatest common factor of two numbers (leading to simplifying (reducing) fractions).
- l. Using the greatest common factor, simplify (reduce) a fraction.
- m. Recognize and change improper fractions to proper fractions and the reverse.
- n. Generate equivalent fractions through multiplication and division using other fractions having the value of 1, completeness (e.g. $2/2$, $3/3$, $4/4$ etc.): $2/3 \times 4/4 = 8/12$ and $8/12$ divided by $4/4 = 2/3$.
- o. Multiply fractions and whole numbers, starting with conversion of the whole number to a fraction with 1 as the denominator (e.g. $2/3 \times 9$ becomes $2/3 \times 9/1$, which equals $18/3$, which can be simplified to 6) and continuing with multiplication of fractions and mixed numbers.
- p. Introduce dividing fractions.
- q. Solve basic word problems using fractions (e.g. "Samuel has 18 chickens. $2/3$ of them are Rhode Island Reds. How many Rhode Island Reds does he have?").
- r. Estimation involving fractions (e.g. what is: $1/2$ of 90?, $2/3$ of 90?).

3. [GEOMETRY AND FORM DRAWING]:

- a. Use the four operations to solve word problems using standard American measurement involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals (with money), and problems that require expressing measurements given in a larger unit in terms of a smaller unit.
- b. Recognize area as an attribute of plane figures and measure areas by counting unit squares.
- c. Work with interwoven form drawings (e.g. Celtic knots).
- d. Basic introduction to degrees as a form of measurement of the relationship between line segments which meet at some point. (E.g. a circle has been given 360 degrees, a number related to time and the approximate number of days in a year as well as the design of an analog clock. If you divide a circle into four equal parts, the measure of each of the four angles created where the dividing lines meet equals 90 degrees. All angles have a measure of between 0 and 360 degrees.) Further developed in 5th and 6th grades.

4. [MEASUREMENT AND DATA]:

- a. Review and practice of all measurement introduced in third grade, including conversions e.g. inches to feet, cups to a quart. Continued practice with word problems.



MPCS Math Learning Goals for Grades 1-8

- b. Reading simple bar and line graphs.
- c. Recognize area as a plane figure and understand the measurement of a unit square as a square with side length of 1 unit. Measure area by counting unit squares (inches, feet, units)
- d. Apply the area and perimeter formulas for rectangles in real world and mathematical problems, with visual representations of units (or tiles) covering an area.

FIFTH GRADE

1. [NUMBER QUALITIES AND RECOGNITION]:

- a. Review and summarize number sense experiences from previous grades (e.g. 4 operations, measurement, fractions).
- b. Move away from concrete representations of number sense (e.g. noting that we still use numbers to count and keep track of things, but sometimes also simply work with numbers: $4/5$ of 20 is 16, but it doesn't have to be 16 oranges or automobiles; it might just be 16).
- c. Generate two numerical patterns using two given rules e.g. add 3 repeatedly, add 6 repeatedly (in separate equations), recognize relationship and pattern that arises.
- d. Introduction to rules of divisibility.

2. [COMPUTATIONS AND PROCEDURES]:

- a. Mastery of long division as introduced and practiced in third and fourth grade, up to two-digit divisors (using rounding of the divisor).
- b. Facility with conversions between mixed numbers and improper fractions (e.g. $49/4 = 49$ divided by 4, which equals 12, with a fractional remainder of $1/4$, so the answer is $12 \frac{1}{4}$. Similarly, $12 \frac{1}{4} = (4 \times 12) + 1$ all over 4, which = $49/4$).
- c. Mastery of Lowest Common Denominator and Greatest Common Factor as employed in finding equivalent fractions and simplifying fractions.
- d. Mastery of four operations with fractions and mixed numbers along with providing visual representations or practical story examples that would represent the problems.
- e. Read, write, and compare decimals to thousandths.
 - i. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$. Recognize a digit in one place is 10 times more or $1/10$ of the number next to it. Recognize patterns in multiplying and dividing by 10, 100, 1000.



MPCS Math Learning Goals for Grades 1-8

- ii. Compare decimals using greater than, less than and equals to.
- iii. Round decimals to the nearest ones, tens and hundreds.
- f. Introduction of four operations with decimals (e.g. 9.6 divided by 3 is 3.2; 3.2 times 3 is 9.6) including long multiplication and division.
- g. Interpret a fraction as division of the numerator by the denominator (e.g. $\frac{3}{4} = 3 \div 4 = 0.75$)
- h. Conversions of fractions to decimal fractions and vice versa.
- i. Basic introduction to percentage as related to decimal fractions and fractions (e.g. $\frac{1}{5}$ of the class has blue eyes. $\frac{1}{5}$ equals 20/100 or .20. We could therefore also say 20 percent of the class has blue eyes). This is further developed in 6th grade with study of business math.
- j. Introduction to simple calculation involving averages. (E.g. if you have four numbers, say 3, 6, 9 and 2, the average all of them may be calculated this way: $3 + 6 + 9 + 2$ all over 4, the number of numbers being averaged, which equals $20/4$, which equals 5, the average.)
- k. Express a whole number in the range 2–50 as a product of its prime factors.
 - i. For example, find the prime factors of 24 and express 24 as $2 \times 2 \times 2 \times 3$
- l. Introduce powers and roots. This may be tied in with square numbers. (E.g. $6 \times 6 = 36$ may be written in a shorthand as $6^2 = 36$. Similarly, $\sqrt{36} = 6$.)
- m. This can also connect with a shorter way of expressing prime factorizations. (E.g. $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ may also be written as $128 = 2^8$, or $1368 = 2 \times 2 \times 2 \times 3 \times 3 \times 19$ may be written as $2^3 \times 3^2 \times 19$.)
- n. Daily mental math practice with all four operations, including fractions, decimals, measurement.
- o. Practice with word problems in all areas including fractions, mixed numbers, measurement, decimals, area and perimeter, etc.

3. [GEOMETRY AND FREEHAND GEOMETRIC DRAWING]:

- a. Draw regular polygons from center to periphery and periphery inward as shaded (plane) figures using both positive and negative space to locate the figures.
- b. Free-hand line drawing of regular polygons leading to more elaborate free-hand drawings (six-division of a circle).



MPCS Math Learning Goals for Grades 1-8

- c. Empirical (pictorial, hands-on) representation of Pythagorean Theorem (no proof until middle school). (Ancient Greece)
- d. Introduction to angles and angle measurement through freehand drawing (compasses introduced in 6th grade). (Ancient Mesopotamia)
- e. Qualities of 3,4,5 right triangle, including measurement of area of right triangle through visual representation. (Ancient Egypt)
- f. Introduction of relationships between intersecting lines and segments (e.g. opposite, complementary, supplementary angles, without proofs). Explore possibilities of relationships between lines/segments and circles.

4. [MEASUREMENT AND DATA]:

- a. Collecting and recording of data related to practical work (e.g. graphing speed, mileage of running, javelin throws, botany -- plant growth, weather -- temperature charts).
- b. Review all measurement from previous years, including continued work on conversions, word problems.
- c. Mastery of perimeter and area calculation for rectangles, introduction of 3rd dimension to calculate volume of rectangular figures (e.g. length x width x height).
- d. Introduction of metric system, with practical activities of measurement (relate to work with decimal fractions).
- e. Conversions within the metric system, including word problems.
- f. Introduction to averages. Finding the average through practical work (e.g. measuring the height of everyone in class and finding the average height, estimates made first).

SIXTH GRADE

1. [COMPUTATIONS AND PROCEDURES]:

- a. Introduction to ratio
 - i. concept of ratio and language to describe it
 - ii. expanding and reducing ratios (with multiplying and dividing e.g. quantities)
 - iii. expressing word problems in ratio form (e.g. A factory produces 1 green racquetball for every 8 red ones. If it produces 16,000 balls in a day, how many of those will be green?)
 - iv. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.



MPCS Math Learning Goals for Grades 1-8

- v. comparison of ratios, inverse proportion (an experience only, perhaps noting, for example, that the shadow of a gnomon becomes shorter with the increasing height of the sun; saving calculation for 7th grade).
- b. Economics/business math
 - i. Discount and unit pricing
 - ii. Banking (practical applications for working with percentages; interest on loans, profit, loss)
 - iii. Money as representation of value; labor as value
 - iv. Basic principles of accounting
 - v. Calculation of simple interest through word problems
- c. Practice and mastery of using all operations with whole numbers, decimals, fractions and mixed numbers (up to level in fractions of multiple step questions with several fractions and operations above and below fraction bar).
- d. Further develop facility in calculation of area, perimeter and simple volumes.
- e. Further practice in computations with more elaborate decimals (e.g. 91.35 divided by 72.5 ; 789.83×7.32) , converting decimals, including repeating ones, to fractions and vice versa).
- f. Further practice with roots and powers.
- g. Compute fluently with multi-digit numbers, finding common factors and multiples.
- h. Further develop concept of distributive property (e.g. in earlier grades it was shown in mental math practice that $7 \times 56 = (7 \times 50) + (7 \times 6)$. This might now be written as just presented or even as $7(50 + 6)$. Likewise, $36 + 8$ can be expressed as $4(9 + 2)$).
- i. Rules for divisibility (e.g. if a number ends in a zero or 5 it is divisible by 5, if in an even number or zero, it is divisible by 2, a number is evenly divisible by three only if the sum of its digits is divisible by 3, etc).
- j. Daily mental math practice.

2. [GEOMETRY AND GEOMETRIC DRAWING]:

- a. Geometrical proofs of sums of angles of a triangle: using cut outs, protractors, simple calculations of these using addition and subtraction of degree measures.
- b. Use of compass and straight-edge in precision work, with many constructions (examples below).
- c. Nature of circles – drawing.
 - i. Radius, diameter, and circumference.



MPCS Math Learning Goals for Grades 1-8

- ii. Division of circle into 3, 4, 6, 8, 12 and 24 parts, hexagons and hexagrams, octagons, -grams, dodecagons, -grams.
- d. Lines
 - i. Parallel lines, concept and construction.
 - ii. Perpendicular lines, concept and construction.
 - iii. Bisecting a line segment.
 - iv. Division of a line into specific segments (e.g. transferring a measure from another figure).
 - v. Line of symmetry
- e. Angles
 - i. Acute
 - ii. Right
 - iii. Obtuse
 - iv. Congruency
 - v. Bisecting angles
 - vi. Alternate and corresponding angles
 - vii. Sum of angles created by intersecting lines
- f. Nature, classes and construction of triangles
 - i. Congruent triangles; the four principal cases for congruency
 - ii. Equilateral
 - iii. Right-angled
 - iv. Isosceles
 - v. Scalene
 - vi. Sum of angles
 - vii. Congruency
- g. Nature, classes and construction of quadrilaterals
 - i. Square
 - ii. Rectangle
 - iii. Rhombus
 - iv. Parallelogram
 - v. Trapezoid
 - vi. Area and perimeter
- h. Advanced Constructions (introduced in 6th, continued in 7th).
 - i. Equiangular spirals
 - ii. Rotations of circles
 - iii. Limacon
 - iv. Cardioid

3. [MEASUREMENT AND DATA]:



MPCS Math Learning Goals for Grades 1-8

- a. **Charts and Graphs (bar graphs, line graphs, pictographs, pie charts).**
 - i. Understanding what type of graph best represents certain data.
 - ii. Drawing and interpreting graphs and charts.
 - iii. Problem solving with graphs and charts.
- b. **Continued practice with measurement and conversions within a system through practical work and word problems.**
- c. **Understanding of simple data analysis: mean, median, mode, range.**

SEVENTH GRADE

1. [COMPUTATIONS AND PROCEDURES]:

- a. **If not covered in sixth grade, introduce basic bookkeeping.**
- b. **Daily mental math practice.**
- c. **Introduce more rules for divisibility as begun in 6th grade.**
- d. **Introduce compound interest, but only compounded annually.**
- e. **Expand on work with roots, going so far as 10th roots: the 10th root of 1024 is 2 (and 2 to its tenth power is 1024).**
- f. **Review both standard American and metric measurement systems until can work with both with ease.**
- g. **Continued practice of conversions within these measurement systems, but not between them except in very general ways (e.g. a meter is about as long as a yard, a kilometer a bit more than half a mile, etc).**
- h. **Introduce negative numbers as numbers with qualities like positive numbers (avoiding too many graphic representations like number lines at first). Use examples such as temperature above/below zero, elevation above/below sea level.**
- i. **Introduce and practice four operations with negative numbers and positive and negative numbers in combination.**
- j. **Understand a rational number as a point on the number line. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.**



MPCS Math Learning Goals for Grades 1-8

- k. Solve real-world and mathematical problems involving the four operations with rational numbers.
- l. Review and mastery of order of operations (use of brackets, etc).
- m. Introduction of irrational numbers (recurring decimals, leading to the value of pi later on).
- n. Continued work with graphic representation of data and its practical application.
- o. Square root algorithm (may be left for 8th grade).
- p. Word problems involving rates (e.g. speed, pay), a continuation of work from 6th grade.
- q. Percentages: increase or decrease, base (e.g. 50 is 20 percent of what number -the base-?).
- r. Ratios
 - i. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
 - ii. Reciprocals of ratios.
 - iii. Proportion of the whole (e.g. cutting a line segment into two smaller segments creates a relationship between the two smaller segments, but these two also relate to the initial whole).
 - iv. Similar figures.
 - v. Direct and inverse proportion.
- s. Introduction to Algebra
 - i. Write, read, and evaluate expressions in which letters stand for numbers.
 - ii. Rates (Continuation and elaboration of formulas introduced in 6th grade).
 - iii. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
 - iv. Summation -- Gauss's Formula
 - v. Expressions:
 - 1. Combining like terms (e.g. variables/unknowns and constants, to simplify expressions).
 - 2. Fractions and decimals as coefficients and constants through practical applications as noted elsewhere (e.g. formulas).

2. [GEOMETRY]:

- a. Golden Mean (e.g. stories, architectural, examples from nature as well as constructions).
- b. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing the drawing at a different scale.



MPCS Math Learning Goals for Grades 1-8

- c. Draw (freehand, with ruler and protractor) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- d. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids, circles, cones.
- e. Pythagorean Theorem.
- f. Areas of geometric shapes through construction and calculation.
- g. Construction of regular pentagon if not done in 6th grade.
- h. Know the formulas for the area and circumference of a circle and use them to solve problems.
- i. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- j. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates.

3. [MEASUREMENT AND DATA]:

- a. Review and elaboration of graphing from 6th grade, use in word problems.

EIGHTH GRADE

1. [COMPUTATIONS AND PROCEDURES]:

- a. Daily mental math practice.
- b. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."*
- c. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. Use proportional relationships to solve multistep ratio and percent problems (e.g. gratuities, commissions, fees, percent increase)
- d. Further development of rational and irrational numbers.
 - i. Know that numbers that are not rational are called irrational. Understand that for every irrational number the decimal expansion eventually repeats, and be able to convert fractions to irrational numbers and vice versa.



MPCS Math Learning Goals for Grades 1-8

- ii. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions.
- e. Simplifying expressions and solving equations.
 - i. Combining like terms
 - ii. Distributing
 - iii. Isolating variable
 - iv. Canceling
- f. Writing and solving variable equations using real numbers.
 - i. Two step (e.g. $5n - 4 = 21$)
 - ii. Multi-step (e.g. $\frac{5n-4}{3} = 7$)
- g. Know and apply the properties of integer exponents to generate equivalent numerical expressions (e.g. $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3}$).
- h. Use square root and cube root symbols to represent solutions to equations of the form $x^3 = p$ and $x^2 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of perfect cubes, know that $\sqrt{2}$ is irrational.
- i. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times more one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is 20 times larger.
- j. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
- k. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has a greater speed.
- l. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .
- m. Solve linear equations in one variable.
 - i. Give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation in the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).



MPCS Math Learning Goals for Grades 1-8

- n. Analyze and solve pairs of simultaneous linear equations.
 - i. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
 - ii. Solve a system of two linear equations by plotting them on a coordinate system and finding the approximate point of intersection (x,y). Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. Determine algebraically whether two lines intersect, and if so at what point (x,y), do not intersect, or are coincident, by solving for one variable in one equation and then substituting that value in place of the same variable in the other equation to determine the point(s) of intersection, if any. (e.g. Given $2x - 3y = -2$ and $4x + y = 24$, a value for y for the second equation may be found, namely, $y = 24 - 4x$. This may be substituted into the first equation, which will then read as $2x - 3(24 - 4x) = -2$, which can be resolved to $14x = 70$ or $x = 5$. Placing 5 as a value for x in either equation give a value to y of 4. The coordinates (5,4) satisfy both equations and therefore name the point of intersection.
 - iii. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
- o. Dimensional analysis
 - i. Conversion between systems of unit measurement, density and rates.
 - ii. Review and practical application of percentages in areas like growth.
- p. Continued work with collection of data including summarization of numerical data, identifying median, mode, mean, deviations from the pattern, etc.
- q. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- r. Introduction of probability. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.
- s. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. [GEOMETRY]

- a. Construct platonic solids.



MPCS Math Learning Goals for Grades 1-8

- b. Review of concepts of area and volume from 6th and 7th grade.**
 - i. Continuation of calculation of area, extended to irregular shapes -- leading to introduction of formulas:**
 - 1. Area of triangles**
 - 2. Area of trapezoid**
 - 3. Area of circle**
 - ii. Volumes of solids.**
 - 1. Prisms**
 - 2. Cylinders**
 - 3. Pyramids**
 - 4. Cones**
 - iii. Word problems with the above**
- c. Surface area of spheres and cones.**
- d. Verify experimentally the properties of rotations, reflections, and translations:**
 - i. Lines are taken to lines, and line segments to line segments of the same length.**
 - ii. Angles are taken to angles of the same measure**
 - iii. Parallel lines are taken to parallel lines**
- e. Further development of Pythagorean Theorem e.g. proof, application in real-world math problems.**
- f. Loci -- the study of curves**



MPCS Math Learning Goals for Grades 1-8

References / Sources

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